optimum vane dimension on the location of the metal shell and the effect of the wire thickness thereupon.

Finally, it is of interest to make a comparison between the structures, with and without vanes, in respect of impedance of the circuit. For numerical appreciation, taking $s = 0$, $N = 3$, $C_r = 6.65$ (beryllia), $\theta = 20°$, $c/a = 2.5$, the optimum vane dimension corresponding to flat dispersion curves is obtained as $(b/a)_{\mathrm{opt}} \approx 1.7$, for the structure with vanes (Fig. 2). For the vaneless structure, all other parameters remaining unchanged, flat dispersion characteristics result only when the shell is brought relatively close to the helix, to an optimum value; in this case, for $(c/a)_{\mathrm{opt}} \approx 1.25$ [7]. Taking these optimized situations, and $\gamma a = 1.6$, the normalized characteristic impedances of the circuit, $2\pi(L/C)^{1/2}(\mu_0/\epsilon_0)^{-1/2}\tan\psi$, with and without vanes, come out to be 0.22 and 0.13, respectively.

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Magnetostatic Forward Volume Wave Propagation—Finite Width

I. J. WEINBERG AND J. C. SETHARES, SENIOR MEMBER, IEEE

Abstract—**The infinite radiation resistance [1] encountered at the low end of the magnetostatic forward volume wave frequency band for a YIG layer of finite width is avoided by employing a physically justifiable low frequency cutoff value higher than that for which radiation resistance would be infinite. Radiation reactance and insertion loss then can be calculated and are found to be relatively insensitive to the choice of the cutoff frequency, except for frequencies very close to cutoff. Beam spreading considerations determine the cutoff frequency.**

I. Theory

By considering Maxwell's equations with the magnetostatic approximation and the permeability tensor in the YIG region [2]

$$\bar{b} = \mu_0[\mu]\bar{h} \tag{1}$$

one obtains the potential function, in non-YIG regions, in the

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I. J. Weinberg is currently in the Air Force AFOSR University Resident Research Program, on leave from the University of Lowell, MA.

J. C. Sethares is with Rome Air Development Center, Hanscom AFB, MA 01731.

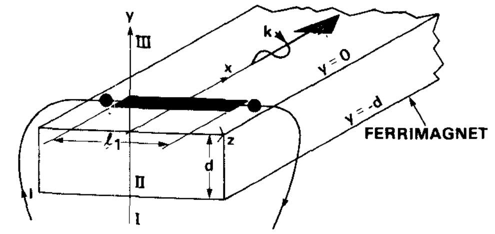

Fig. 1. Transducer geometry.

form

$$\psi = \int_{-\infty}^{\infty} e^{-ikx}\cos\frac{n\pi}{l_1}z\left(Ae^{\bar{k}y} + Be^{-\bar{k}y}\right)dk, \qquad n = 0,1,2,3\cdots \tag{2}$$

where k is the wave number in the x direction, l_1 is the strip width (Fig. 1), n represents the width mode and

$$\bar{k} = \sqrt{k^2 + (n\pi/l_1)^2}, \qquad n = 0,1,2,3,\cdots. \tag{3}$$

The components of $\bar{h}$ are found from the derivative of ψ. For $n = 0$, we have the infinite width case. For odd n, the potential vanishes at the strip ends $z = \pm l_1/2$.

For the YIG region, the potential function is taken from the basic equations to be in the form

$$\psi = \int_{-\infty}^{\infty} e^{-ikx}\cos\frac{n\pi}{l_1}z\left(A\cos\alpha\bar{k}y + B\sin\alpha\bar{k}y\right)dk, \qquad n = 0,1,2,3,\cdots \tag{4}$$

where, for forward volume waves [3]

$$\alpha = \left[-\left(1 + \frac{\gamma^2 HM}{\gamma^2 H^2 - f^2}\right)\right]^{1/2} \tag{5}$$

and

$$\gamma = 2.8\ \mathrm{MHz/oe}, \quad M = 1750\ \mathrm{oe}, \quad f = \omega/2\pi \tag{6}$$

and H is the biasing field magnitude.

Considering the case of no ground planes, we determine the constants in (2) and (4) for the three regions (Fig. 1) by requiring ψ to be finite at $y = \pm\infty$, b_y to be continuous at $y = 0$ and $y = -d$, h_x to be continuous at $y = -d$ and, for a given current distribution

$$h_{x\mathrm{III}} - h_{x\mathrm{II}} = J_z(x), \qquad \text{at } y = 0. \tag{7}$$

Application of boundary conditions yields the dispersion relation (see [3, eq. 17])

$$\left[(\alpha^2 - 1)\sin\alpha\bar{k}d - 2\alpha\cos\alpha\bar{k}d\right] = 0 \tag{8}$$

or

$$\bar{k} = \frac{1}{\alpha d}\tan^{-1}\frac{2\alpha}{\alpha^2 - 1} + \frac{m\pi}{\alpha d}, \qquad m = 0,1,2,3\cdots. \tag{9}$$

There are an infinite number of thickness solution modes, corresponding to the value of m, with $m = 0$ giving the fundamental mode.

By utilizing (8) and integrating (7) in the x direction, $-\infty$ to ∞, in the usual manner and integrating in the z direction, $-l_1/2$ to $l_1/2$, we obtain all constants in (2) and (4). Appropriate premultiple factors are used in these integrations.

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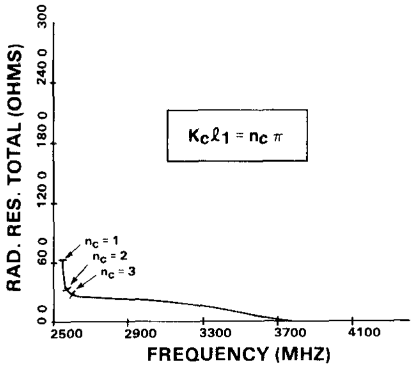

Fig. 2. Radiation resistance.

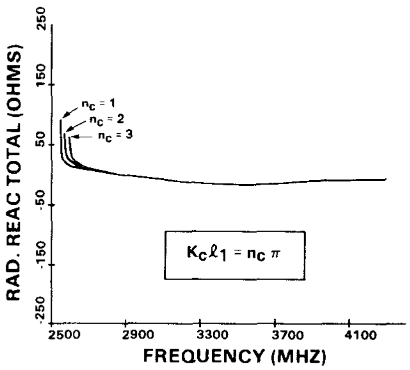

Fig. 3. Radiation reactance.

We then obtain expressions for ψ and the components of h in the three regions. We then perform the usual contour integrations to find the field equations [3].

The magnetostatic wave power obtained from [1]

$$P=\frac{1}{l_1}\int_{-l_1/2}^{l_1/2}\left[\int_{-\infty}^{\infty}-i\frac{\omega}{2}\psi^* b_x\,dy\right]dz \tag{10}$$

becomes, after utilizing (1) and the dispersion relation

$$P^s=\left(\frac{\bar{k}_s}{k_s}\right)^2\frac{\mu_0\omega\tilde{J}^2(k_s)}{4k_s d(\alpha^2+1)}\begin{Bmatrix}1, & n=0\\ 0, & n=2,4,6,\cdots\\ \frac{1}{2}\left(\frac{4}{n\pi}\right)^2, & n=1,3,5,\cdots\end{Bmatrix}. \tag{11}$$

$\bar{k}_s$ is obtained from the dispersion relation (9), k_s is obtained from (3), $\tilde{J}(k_s)$ is the Fourier transform of $J_z(x)$ and is obtained as before [3] for a flat current distribution. Note that even n produces no solution modes.

The radiation resistance for one strip is, for unity current

$$R^s=2P^s \tag{12}$$

and the total radiation resistance is twice R^s since the results are the same for each of the two waves. S refers to propagation direction.

When $n=0$, the infinite width case, the lower cutoff frequency is taken as γH where k_s vanishes and the power and radiation resistance are finite there from (11) because of the infinite behavior of α there.

For $n>0$, finite width, the power and radiation resistance would become infinitely large from (11) at the lower cutoff frequency where k_s is zero since this frequency is greater than γH and α is not infinite there. However, the following is justification for taking the lower cutoff frequency to be larger than this value at which k_s is zero. When magnetostatic wavelengths λ, where $\lambda=2\pi/k$, are small compared to the transducer aperture l_1, a well-collimated beam is formed and beam spreading is negligible. When magnetostatic wavelengths approach or exceed l_1, beam width increases, permitting a smaller portion of the total energy leaving the transducer to reach the output. This means a lower radiation resistance. In the limit, $l_1/\lambda\to 0$, one would expect the radiation resistance to vanish. Thus, instead of taking the lower cutoff frequency to be at which k_s is zero and power and radiation resistance are infinite from (11) and (12), we take the lower cutoff frequency to be at which

$$k_{sc}=n_c(n\pi/l_1) \tag{13}$$

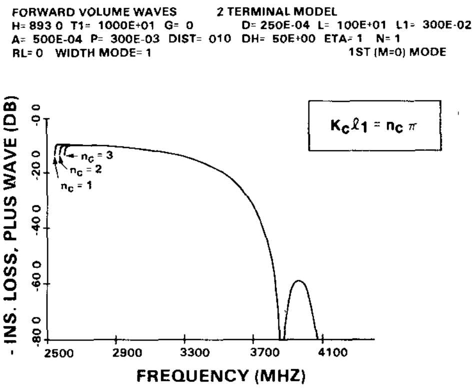

Fig. 4. Insertion loss

where n_c is a constant determined so that $\lambda\leqslant l_1$. Appreciable beam spreading occurs when $l_1\sim\lambda$ or when (13) is satisfied for $n_c\sim 2$, when $n=1$. When $n>1$, n_c is found to be less than 2.

In Fig. 2, we show the total radiation resistance for the first width mode, $n=1$, for $n_c=1$, 2, and 3. Here $H=893$ and $d=25$ μm.

In Figs. 3 and 4, we show the total radiation reactance obtained by Hilbert transform and the insertion loss [3] for one wave, for each of the three values of n_c.

We observe that, except at the very beginning of the bandwidth, the results are the same for the three n_c values so that the lower cutoff frequency value is not critical, as long as it is sufficiently higher than that for which k vanishes.

II. Conclusion

We have examined magnetostatic forward volume waves for finite width and have shown that it is physically justifiable to remove the end of the bandwidth where k_s is zero so that the radiation resistance is not infinite and the radiation reactance and insertion loss may be calculated.

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